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Evaluation of Different Modeling Approaches for Estimating Total Bole Volume of Hispaniolan Pine (*Pinus occidentalis* Swartz) in Different Ecological Zones

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Abstract: *Pinus occidentalis* (Swartz) is the primary timber species in the Dominican Republic (DR). Despite its economic importance, studies conducted on this species are scarce, making it difficult to estimate current inventory levels. This study aims to enhance the accuracy of estimating the total bole volume of *P. occidentalis* in different ecological zones (EZs) within La Sierra, evaluating and comparing two established volume equations—combined variable (CV) and Schumacher and Hall (S&H) across nine modeling variants. An indicator variables analysis determined the necessity of distinct equations for two EZs. Fitting included both linear and nonlinear models. Our comprehensive statistical analysis included goodness-of-fit metrics to evaluate each model variant's performance rigorously. The second modeling variant (SH02) for the SH equation was most effective in the Dry Ecological Zone, showing superior performance in both the fitting and validation phases. Similarly, the third modeling variant (SH03) for the SH equation emerged as the best fit for the Combined Intermediate and Humid Ecological Zones, achieving the lowest overall ranking sum among tested variants. SH02 and SH03 provide reliable and precise volume estimations, allowing for the optimization of forestry management practices for *P. occidentalis* trees. The SH models outperformed the CV model variants' consistency in parameter estimation. This tailored approach ensures more accurate volume predictions, which is crucial for sustainable management and conservation efforts.

Keywords: indicator variables analysis; ecological zones; modeling variants; goodness-of-fit; volume estimation; forestry management

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1. Introduction

Accurate measurements of tree dimensions like diameter, height, stem form, and volume are not just academic exercises but crucial for estimating forest attributes. This research provides practical tools like individual-tree and species–specific volume equations, which are necessary to predict inventory levels and ensure long-term wood yields. Volume equations use diameter at breast height (DBH) and height (H) to calculate stem volume, providing reliable estimates of tree volumes by accounting for size and shape variations [1]. These practical tools are essential in forestry practices, empowering forest managers and timber industry professionals with accurate and reliable data.

Stem volume models estimate timber volume, biomass, and carbon sequestration potential for individual trees and larger areas. They are useful in forest management planning and growth simulations, driven by the need for the accurate estimation of tree bole volume. Because the same regression may not be equally suitable for predicting total volume estimates in different ecological conditions [1], specific details and techniques for developing statistical models for stem volume estimation may vary.

The history of excurrent tree bole volume estimation dates back several decades [2]. Various statistical techniques can be used to develop stem volume models, including linear regression, nonlinear regression, mixed-effects procedures, and machine learning algorithms [2]. The choice depends on the data set's underlying assumptions and characteristics [2]. They are developed in a two-step process: model fitting and validation. The model is fitted to the data set using a statistical technique and then validated using an independent data set. Evaluation uses measures such as mean squared error, root mean squared error, bias, and the coefficient of determination.

The most common statistical procedure for developing volume tables are ordinary least squares (OLS) regression analyses, which relate bole volume to explanatory variables such as DBH, H, and sometimes stem form. When dealing with biological data, the constant variance assumption is often violated whenever a direct measure of stem content is used as the dependent variable in a regression equation [3]. If the assumption of constant variance is unmet, the equation must be weighted by a factor proportional to the standard deviation of the dependent variable.

Weighted least squares can be used when the ordinary least squares assumption of constant variance in the errors is violated. It will produce a new regression model which results in the dependent variable having constant variance [3]. In the weighted linear regression model, each observation is assigned a weight W_i. The weighted sum of squared residuals, $Q = \sum_{i=1}^{n} W_i \times (\hat{\varepsilon}_i)^2$ is minimized.

Assumptions related to linear and nonlinear statistical models involve linearity, independence, homoscedasticity, normality, and zero mean of residuals [4]. Additionally, nonlinear models must have a correct expectation function [4]. These assumptions guide the development and application of these equations in forestry practices, and their violation may lead to biased predictions and incorrect inferences.

The scarcity of data on volume models for tropical and sub-tropical tree species negatively impacts the accuracy of estimating tree volume for both research and operational purposes [5]. Even when developed, stem volume models should be periodically updated to account for changes in forest structure, climate conditions, or management practices to ensure accurate predictions and relevant decision support. *P. occidentalis* is the main timber species in the DR, growing on approximately 302,500 hectares and comprising approximately 95% of all the timber harvested [6]. Despite its economic importance, estimating inventory levels and accounting for harvested volume is difficult because no standardized system is used for volume appraisal and inventory purposes.

The Schumacher and Hall [7] equation (SH) has been widely used in forestry to estimate tree bole volumes. Other researchers have shown that this equation and the Combined Variable equation developed by Bennett et al. [8] provide accurate results for estimating tree volumes, good performance in graphical analysis, and reliable estimates without bias [9].

The objectives of this study were as follows: (1) to fit two commonly used volume equations, the combined variable (CV) and the SH equations with nine different modeling alternatives, to estimate the total bole volume content of *P. occidentalis* trees growing in three ecological zones (EZ) within La Sierra, DR; (2) to evaluate goodness-of-fit statistics of these nine modeling alternatives in each EZ; (3) to conduct and indicator variable analysis to determine if separate equations were needed for each EZ; and (4), based on the ranking of performance in terms of accuracy and precision, recommend the bes alternative for estimating total bole volume in individual *P. occidentalis* trees.

2. Materials and Methods

The study area is in the north-central part of Cordillera Central, DR, covering approximately 1800 km^2 (Figure 1). Even-aged natural stands of *P. occidentalis* are located within three ecological zones according to the Holdridge [10] classification: Subtropical Dry Forest (Dry Zone), Subtropical Humid Forest (Intermediate Zone), and Subtropical Very Humid Forest (Humid Zone). Average elevations above sea level are 500, 650, and 800 m, respectively [11]. The climate varies depending on the altitude and precipitation.

The average annual temperature is between 12 °C and 24 °C. These forests usually develop in shallow, carbonate, lateritic, low-producing soils with rugged topography.

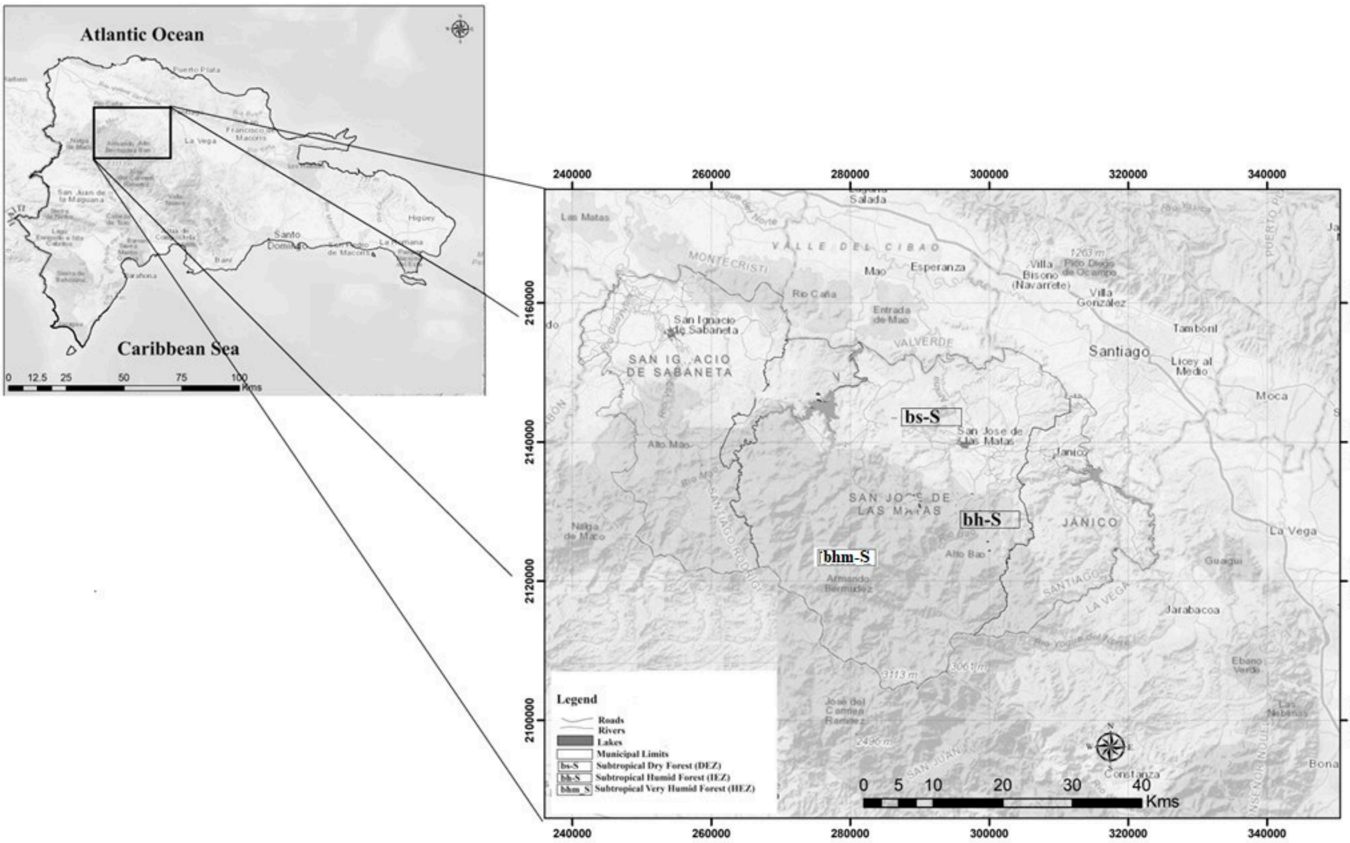

Figure 1. Study area within the La Sierra region in the north-central portion of Cordillera Central, Dominican Republic.

2.1. Tree Data Sets

Three data sources were used for fitting total bole volume outside bark (V_{SOB}) models, one for each ecological zone. Trees from either dominant, intermediate, or overtopped classes in each zone were selected for destructive sampling, given that their phytosanitary status would allow it. After recording DBH outside bark at 1.30 m from the ground, measured with a diameter tape (0.1 cm precision), trees were felled as close to the ground as possible, and H was measured with open-reel tape (0.1 cm precision). Besides DBH, other diameters were recorded at every meter along the bole from the base (10 cm above ground) up to the apex. Stem analysis data for model fitting purposes included 37 trees from the Subtropical Dry Forest, 48 from the Intermediate Zone, and 72 from the Humid Zone. In addition, an independent data set collected from each zone was available to validate the resulting best total V_{SOB} model. These data included 85, 90, and 75 independently measured *P. occidentalis* trees for the Dry, Intermediate, and Humid Zones. Table 1 showcases descriptive statistics for both the fitting and validation data set.

Table 1. Descriptive statistics of tree measurements in each ecological zone for the fitting and validation data sets used in the study.

Variable	Ecological Zone	*n*	Mean	Std Dev	Minimum	Maximum
			Fitting Data Set			
Diameter (cm)	Dry Zone	37	22.13	6.57	11.50	42.00
	Intermediate Zone	48	32.63	8.34	16.50	53.50
	Humid Zone	72	31.18	5.18	21.50	46.50
Height (m)	Dry Zone	37	16.49	2.92	10.30	24.00
	Intermediate Zone	48	19.78	2.78	14.00	25.10
	Humid Zone	72	24.65	4.76	14.50	35.00
Volume (m^3)	Dry Zone	37	0.34	0.22	0.06	1.10
	Intermediate Zone	48	0.66	0.24	0.32	1.30
	Humid Zone	72	0.98	0.57	0.20	2.76
			Validation Data Set			
Diameter (cm)	Dry Zone	85	21.32	7.95	8.00	42.10
	Intermediate Zone	90	27.26	8.20	11.00	54.20
	Humid Zone	75	30.06	7.87	10.60	50.10
Height (m)	Dry Zone	85	16.33	4.46	7.30	26.10
	Intermediate Zone	90	19.96	4.06	10.10	27.80
	Humid Zone	75	20.65	3.40	9.40	27.40
Volume (m^3)	Dry Zone	85	0.35	0.25	0.10	1.23
	Intermediate Zone	90	0.55	0.35	0.12	2.22
	Humid Zone	75	0.64	0.36	0.11	1.81

n: number of observations; Std Dev: Standard Deviation.

2.2. Data Exploration

To visualize the form of the relation between the dependent variable V_{SOB} and the chosen explanatory variables DBH and total tree height (H), we proceeded to merge corresponding data from all three EZs and plot the V_{SOB} against DBH first, and later against the combined variable diameter square times total tree height, D^2H.

2.3. Approaches to Individual Tree Volume Prediction

2.3.1. Indicator Variables Analysis

An indicator variable analysis was conducted to check if three different equations were necessary for each EZ. We searched for statistically significant differences in the intercept and slope parameters in a regression equation fitted to the data for all three EZs, employing a single combined effect variable (D^2H) and indicator variables. A statistically significant test for the intercept and/or the slope for two zones would indicate that each would require different equations. The following model was fitted to indicator variables and the continuous variable D^2H in the indicator variable analysis:

$$V_{ib} = (\beta_0 + \delta_1 Z_1 + \delta_2 Z_2) + \left(\beta_1 D^2H + \delta_3 Z_1 D^2H + \delta_4 Z_2 D^2H\right) + \varepsilon \tag{1}$$

where

- $D,\ H,\ V_{ib},\ \varepsilon$ are as previously defined;
- Z_1, Z_2 are dichotomous variables;
- $\beta_0,\ \beta_1,\ \delta_1,\ \delta_2,\ \delta_3,\ \delta_4$ are the parameters to be estimated.

2.3.2. Total Bole Volume Model Fitting

Observed outside-bark bole volume (m^3) computations were performed using Smalian's formula [12] for each 1 m section from the base to the apex, except for the last portion where the cone formula was used. The outside-bark volume computed for each section was then summed up for each tree to calculate V_{SOB}.

To estimate/predict V_{SOB} of *Pinus occidentalis* individual trees, we choose two models.

The combined variable equation (CV):

$$V_{SOB} = \beta_0 + \beta_1 D^2 H + \varepsilon \tag{2}$$

- Schumacher and Hall's [7] equation (SH):

$$V_{SOB} = \beta_1 \times D^{\beta_2} H^{\beta_3} \times \varepsilon \tag{3}$$

where

- V_{sob} = total stem volume content outside bark (m^3);
- D = normal diameter at 1.30 m from the ground outside bark (cm);
- H = total tree height (m);
- Ln = natural logarithm;
- ε = error term;
- $\beta_0,\ \beta_1,\ \beta_2$ = coefficients to be estimated.

We fitted five variants to our model [2] and four variants to our model [3], including linear, nonlinear, and weighted regression estimation techniques. The ordinary and weighted least squares method fitted the five Model [2] variants. These variants are as follows:

a. (CV01) Original equation
b. Weighted linear regression using four different weights:
    i. (CV02) Weight 1 = 1/fitted values from the original linear regression between the dependent variable "observed volume" (Vol) and the predictor normal diameter squared times total tree height (D^2H);
    ii. (CV03) Weight 2 = 1/fitted value resulting from fitting the absolute values of original residuals against the fitted values of original combined variable regression;
    iii. (CV04) Weight 3 = 1/fitted value resulting from fitting squared values of original residuals against the fitted values of original combined variable regression;
    iv. (CV05) Weight 4 = 1/D^{2c}, where the variance of ε is assumed to be proportional to D^{2c} [13].

Where

- CV0i = variant identification code for model [2];
- C = exponent to be assumed or estimated.

The four variants from model [3] were fitted by the ordinary least squares and nonlinear methods, using multiple regression techniques between the dependent variable, V_{SOB}, and the predictor variables, DBH and total tree height (H).

i. (SH01) De-transformation of the logarithmic conversion ($Ln(V_{SOB}) = Ln(\beta_0) + \beta_1 LnD + \beta_2 LnH + Ln\varepsilon$), solved by employing linear regression and correcting for bias. The correction is achieved by adding one-half of the estimated variance from the fitted regression before exponentiation [14]. The resulting expression is as follows:

$$V_{SOB} = e^{(\hat{\beta}_0 + \hat{\beta}_1 \times ln(D) + \hat{\beta}_2 \times ln(H) + \frac{\hat{\sigma}^2}{2})} \tag{4}$$

where

- V_{SOB} = corrected estimate of the stem volume outside bark;
- $\hat{\mu}$ = mean volume outside bark estimated in log scale;

- $\frac{\hat{\sigma}^2}{2}$ = half-estimated variance in log scale.

ii. (SH02) Nonlinear SH (model [2]) version.
iii. (SH03) Nonlinear weighted version SH version assuming exponent $c = 2$;
iv. (SH04) Nonlinear weighted SH version with modeled variance (exponent c), where the variance of ε is assumed to be proportional to D^{2c} [13].

2.3.3. Statistical Analysis

Weighted linear and nonlinear least squares were used to maximize the efficiency of parameter estimation. All tests on the full model parameters were conducted at $\alpha = 0.05$. Data analysis and model development procedures were performed using lm, nls, and nlme commands in RStudio [15] to obtain parameter estimates for V_{SOB}.

Starting values of the coefficients for nonlinear and weighted nonlinear variants were obtained by applying ordinary least squares to the log-transformed data, ensuring faster iteration. Even though log-transformed and nonlinear models are not mathematically equivalent, the coefficients of the former estimated by multiple regression may serve as starting values for the algorithm that estimates the coefficients of the latter [13].

To estimate coefficient c for the weighting of the observations in variants CV05 and SH04, observations were divided into five DBH classes containing approximately the same number of observations. Then, we calculated the standard deviation of V_{SOB} in each D class. Following a methodology from Picard et al. [13], we plotted the standard deviation of V_{SOB} against the median DBH in each of the five classes in the log scale. The five points on the plot should be roughly aligned along a straight line to confirm that the power model was appropriate for modeling the residual variance. If that were the case, we would proceed to fit a linear regression of the log of the standard deviation of V_{SOB} on the log of the median D for each class. The slope of such regression corresponds to exponent c. The standard deviation of the stem volume would be approximately proportional to D^{2C}, and a weighting of the observations would be inversely proportional to it.

Statistical analyses performed on model [3] variant SH01 included initially working on the log-transformed data and fitting an ordinary least squares (OLS) multiple regression of ln(V) against ln(D) and ln(H), and then transforming to original units the coefficient estimates of this initial equation and correcting it for bias by adding one-half of the estimated variance from the fitted regression before exponentiation.

2.3.4. Evaluation Criteria

Model Validation and Goodness of Fit Statistics

The goodness-of-fit statistics used to determine how well the regression functions fitted the sample data were (1) root mean square error (*RMSE*); (2) *Bias*; (3) the sum of squared relative residuals (*SSRR*); (4) the residual variance estimator (*RVE*), and (5) Akaike Information Criteria (*AIC*), used only in the fitting phase to compare different models and balance the goodness of fit with model complexity.

The "validation" statistics used to determine how well the regression functions performed on the independent data representing the population were (1) *RMSE*, (2) *Bias*, (3) *SSRR*, and (4) *RVE*. The best model for total bole volume in each of the three zones was selected based on the ranking of these evaluation criteria. We also considered the significance of parameter estimates [13,16]. The computational formulas for the goodness-of-fit statistics are as follows:

$$AIC = -2\, ln\ell\left(\hat{\theta}\right) + 2q \tag{5}$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(V_{sob} - \hat{V}_{sob}\right)^2}{n-p}} \tag{6}$$

$$Bias = \frac{\sum_{i=1}^{n}\left(V_{sob} - \hat{V}_{sob}\right)}{n} \tag{7}$$

$$SSRR = \sum_{i=1}^{n} \left[\frac{V_{sob} - \hat{V}_{sob}}{V_{sob}} \right]^2 \quad (8)$$

$$RVE = \frac{n}{n-p-1} \left(1 - R^2 \right) S_Y^2 \quad (9)$$

where

ℓ: is the model's likelihood;
p: is the number of free parameters estimated;
V_{sob}: is the observed stem wood volume outside bark;
$\hat{V}_{sob}$: is the estimated stem wood volume outside bark;
n: is the total number of observations;
S_Y^2: is the empirical variance of the response variable.

Ranking of Models

To rank the nine modeling variants tested in each EZ, fit and validation goodness-of-fit statistics values were ranked. Rank valued No. 1 was linked to the best value for each of these statistics, rank No. 2 the second best, and so on. The overall rank for each model variant was determined by summing the ranks for the various goodness-of-fit statistics for the total volume and then choosing the lowest sum as the best model variant.

Residual and Quantile-Quantile Plot Graphs

Scatterplots were constructed to check regression assumptions for the best-ranked model variants in the Dry Zone and the Combined Intermediate and Humid Zones to check that the hypotheses assumed for the residuals were satisfied.

The hypothesis that the residuals were independent has already been satisfied due to the sampling plan adopted. The constant variance hypothesis of the residuals was visually checked by plotting the cluster of points for the residuals $\varepsilon_i = Vol_{SOB-}\hat{Vol}_{SOB}$ in function to the predicted values. The hypothesis that the residuals are normally distributed was visually inspected with the quantile–quantile graphs, plotting the residuals' empirical quantiles against the theoretical quantiles of the standard normal distribution. To further assess how well predictions from model variants align with the actual data, we plotted observed-versus-predicted stem volume values and observed volumes versus predictor variables.

3. Results

3.1. Data Exploration

On average, sampled trees were smaller in terms of DBH, H, and V_{SOB} in the Dry, and largest in the Humid. DBH ranged from 11.50 to 42.00 cm in the Dry Zone, 16.50 to 53.50 cm in the Intermediate Zone, and 21.50 to 46.50 cm in the Humid Zone. Following the same zone order, H from 10.30 to 24.00 m, 14.00 to 25.10 m, and 14.50 to 35.00 m. Volume outside bark ranged from 0.06 to 1.10 m^3, 0.32 to 1.30 m^3, and 0.20 to 2.73 m^3, respectively (Table 1).

The plotted points in the left panel of Figure 2 show that the relationship between DBH and volume is not linear. Volume variance increases as DBH increases. The relationship between V_{SOB} and the combined effect variable D^2H, shown on the right side of Figure 2, shows that the relationship between these two variables is linear; but, as before, the volume variance increases with D^2H.

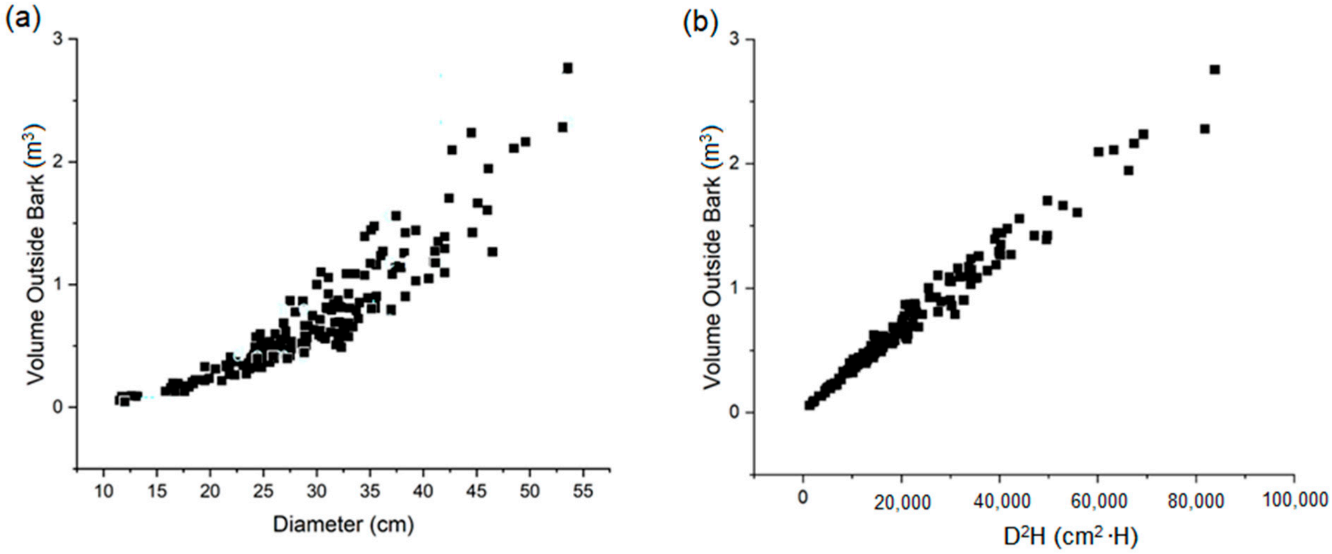

Figure 2. Scatterplots of the relation between V_{SOB} and DBH (**a**) and D^2H (**b**). On the left, the relationship is nonlinear and presents heteroscedasticity. On the right, the relationship is linear, but heteroscedasticity persists.

3.2. Indicator Variables Analysis (IVA)

The results of the Indicator variable analysis show that the Humid and Dry Zones have different intercepts and the same slope. Humid and Intermediate Zones have the same intercept and slope, and Dry and Intermediate Zones have the same intercept but different slopes (Table 2). Therefore, the Humid and Intermediate Zones can have the same equations to estimate the V_{SOB} of individual *P. occidentalis* trees, while the Dry Zone requires its own equations. Tree stem form has been observed to change in different zones with variation attributable to environmental factors [1]. Based on these indicator variable analysis results, we combined the observed data from the Humid and Intermediate Zones and fitted the tested models to the combined data.

Table 2. Statistical test results for the intercept and slope coefficients from the indicator variable analysis of the dependent variable stem volume outside bark in the three ecological zones.

Zone	Intercept	Slope
Humid versus Dry	Different: *p*-value = 0.0277	Same: *p*-value = 0.1414
Humid versus Intermediate	Same: *p*-value = 0.4851	Same: *p*-value = 0.974
Dry versus Intermediate	Same: *p*-value = 0.104	Different: *p*-value = 0.0294

Merged fitting data from the Humid Zone and Intermediate Zone totaled 121 observations. DBH ranged from 16.50 to 53.50 cm, H ranged from 14.00 to 35.00 m, and Vol_{Sob} ranged from 0.20 to 2.76 m^3. Respective averages and sample standard deviations (within parenthesis) were 31.77 (±7.05) cm, 22.67 (±4.73) m and 0.84 (±0.48) m^3. After merging data from both zones, the validation data set had 165 observations with DBH ranging from 10.60 to 54.20 cm, H ranging from 9.40 to 27.80 m, and Vol_{Sob} from 0.11 to 2.22 m^3. Respective averages and sample standard deviations were 28.53 (±8.15) cm, 20.27 (±3.78) m, and 0.59 (±0.36) m^3.

3.3. Total Bole Volume Model Fitting in the Dry Ecological Zone and the Combined Intermediate and Humid Ecological Zone

The SH02 variant, fitted by nonlinear least square (NLS) procedures, was ranked number one for estimating V_{SOB} *P. occidentalis* in the Dry Zone, while in the Combined Intermediate and Humid Zone, the best variant was SH03, which was fitted by weighted nonlinear least square (WNLS) procedures. Based on the ranks of the goodness-of-fit statistics criteria applied to the data fitting and validation phases, these results indicate the superiority of the SH model variants in this study. The fitting and validation statistic values, along with rank sums and the overall ranks for each modeling variant in each EZ, are presented in Tables 3 and 4.

Considering both the fitting and validation phases, SH02 in the Dry Zone was ranked No. 1 in terms of RMSE (0.0146) and RVE (0.0002) and No. 2 in AIC criteria (−199.86) in the fitting phase (Table 3). Likewise, it was ranked No. 1 in terms of RMSE (0.0702), Bias (−0.0099), and RVE (0.0030) in the validation phase. Percent bias calculated as $\left[V_{SOB} - \hat{V}_{SOB}/V_{SOB}\right] \times 100$ in the fitting phase indicated that SH02 tends to slightly overestimate V_{SOB} by approximately −0.059%. Percent RMSE was calculated at 4.77% for SH02, suggesting that predictions deviate by about 4.77% from the mean of the actual values. We consider these values to be adequate for estimating V_{SOB} for the individual trees of *P. occidentalis* in this EZ.

SH02 had an overall sum rank of 31, being the lowest ranked of all volume variants tested in the Dry Zone. It was followed by SH04, the fourth variant of model (3) with a sum rank of 33 and fitted by WNLS procedure, where exponent "c" in the variance model ($Var(\varepsilon) = (kDBH^c)^2$) was a parameter that needed to be estimated.

SH03 in the Combined Intermediate and Humid Zone was ranked No. 1, achieving the lowest sum in the ranking (23) among the nine variants tested in this combined EZ (Table 4). For weights, the conditional standard deviation of V_{SOB} derived from DBH was proportional to DBH^4. In the fitting phase, SH03 first regarded AIC criteria (−311.66). Similarly, it was ranked No. 1 in terms of RMSE (0.0943), Bias (−0.0614), and RVE (0.0059) in the validation phase. Percent bias calculated as $\left[V_{SOB} - \hat{V}_{SOB}/V_{SOB}\right] \times 100$ in the fitting phase indicated that SH03 tends to underestimate V_{SOB} by approximately 0.025% on average. Percentage RMSE was calculated at 8.96%, suggesting predictions that deviate by about 8.96% from the mean of the actual values for estimating V_{SOB} for the individual trees of *P. occidentalis* in the Combined Intermediate and Humid Zone. Regarding the sum rank, SH03 was followed by modeling variant SH01, the unweighted nonlinear version of the SH function, with a sum rank of 35.

The parameterization of SH02 and SH03, respectively, are as follows:

$$V_{SOB-DEZ} = 0.00005815 \times D^{1.7802} H^{1.0787}$$

$$V_{SOB-CIHEZ} = 0.00005669 \times D^{1.7019} H^{1.0786}$$

Although different fitting procedures estimated them, the intercept and coefficient corresponding to the Dry Zone and Combined Intermediate and Humid Zone tree height are very similar. The intercepts differed by 2.54%, and the tree height (H) coefficients differed by 0.009%. The coefficients for D differed by 4.49%.

In checking regression assumptions, our plots show no curvature. However, there is one point beyond the cutoff limit of two standard deviations in the top left panel corresponding to the Dry Zone. We assumed modeling variants SH02 and SH03 do not seriously contradict the constant variance assumption. Even though the quantile–quantile plots of the residuals appear to have a slight structure in both zones, most of the points are aligned along a straight line and remain relatively consistent across all the levels of the predictor variables, indicating that the constant variance assumption is sustained.

All nine variants tested from models (2) and (3) in the Dry and Combined Intermediate and Humid ecological zones resulted in parameter estimates that are statistically significant and logically consistent for outside-bark volume estimation (Tables 5 and 6). The estimates for intercept, interpreted as the average value of the total stem volume accumulated by a tree until it reaches breast height (when DBH is zero and total height is 1.3 m) were positive and significantly different from zero. All other parameters were consistent in terms of sign and magnitude.

Table 3. Goodness-of-fit statistics and rank (ranking value in parenthesis below corresponding statistic) were obtained in the fitting and validation stage of five variants of the combined variable equation and four of the Schumacher and Hall [7] model to estimate the stem volume outside bark of *P. occidentalis* trees in the Dry Zone within La Sierra, Dominican Republic.

		Fit Statistics					**Validation Statistics**				**Ranking**	
Model	**Variant Code**	**RMSE (Rank)**	**BIAS (Rank)**	**SSRR (Rank)**	**RVE (Rank)**	**AIC (Rank)**	**RMSE (Rank)**	**BIAS (Rank)**	**SSRR (Rank)**	**RVE (Rank)**	**Sum Rank**	**Overall Rank**
Model (2): Effect Variable D^2H	CV01	1.61E−02 (5)	3.87E−19 (1)	1.06E−01 (9)	2.82E−04 (9)	−1.95E+02 (8)	7.92E−02 (4)	−1.32E−02 (5)	5.98E+00 (1)	3.52E−03 (4)	46	6
	CV02	1.62E−02 (6)	9.96E−19 (2)	9.66E−02 (6)	2.80E−04 (5)	−2.07E+02 (4)	8.11E−02 (5)	−1.28E−02 (3)	6.41E+00 (2)	3.63E−03 (5)	38	3
	CV03	1.62E−02 (7)	−2.41E−04 (6)	9.67E−02 (7)	2.80E−04 (6)	−2.06E+02 (5)	8.21E−02 (7)	−1.37E−02 (6)	6.42E+00 (3)	3.71E−03 (6)	53	7
	CV04	1.63E−02 (9)	−5.93E−04 (8)	9.57E−02 (4)	2.81E−04 (8)	−2.12E+02 (1)	8.31E−02 (8)	−1.38E−02 (7)	6.59E+00 (5)	3.77E−03 (8)	58	8
	CV05	1.63E−02 (8)	−6.83E−04 (9)	9.62E−02 (5)	2.81E−04 (7)	−2.11E+02 (2)	8.32E−02 (9)	−1.42E−02 (8)	6.51E+00 (4)	3.77E−03 (9)	61	9
Model (3): Effect Variables D, H	SH01	1.48E−02 (4)	2.70E−04 (7)	9.55E−02 (1)	2.49E−04 (4)	−1.08E+02 (9)	7.36E−02 (3)	−1.30E−02 (4)	7.20E+00 (6)	3.19E−03 (3)	41	4
	SH02	1.46E−02 (1)	−1.81E−04 (5)	9.85E−02 (8)	2.39E−04 (1)	−2.00E+02 (6)	7.02E−02 (1)	−9.89E−03 (1)	7.28E+00 (7)	2.97E−03 (1)	31	1
	SH03	1.47E−02 (3)	−6.23E−06 (3)	9.56E−02 (2)	2.43E−04 (3)	−2.11E+02 (3)	8.14E−02 (6)	−1.89E−02 (9)	7.33E+00 (8)	3.71E−03 (7)	44	5
	SH04	1.47E−02 (2)	−2.54E−05 (4)	9.56E−02 (3)	2.42E−04 (2)	−1.97E+02 (7)	7.15E−02 (2)	−1.02E−02 (2)	7.33E+00 (9)	3.06E−03 (2)	33	2

CV01: Combined Variable function first variant; CV02: Combined Variable function second variant; CV03: Combined Variable function third variant; CV04: Combined Variable function fourth variant; CV05: Combined Variable function fifth variant; SH01: De-transformed S&H Model with Bias Corrected variant; SH02: S&H Non-linear Model variant; SH03: S&H Weighted Non-linear Model Assuming Exponent C variant; SH04: S&H Weighted Non-linear Model Modeling Exponent C variant; H: Total Tree Height.

Table 4. Goodness-of-fit statistics and respective ranking obtained in the fitting and validation stage of the five variants of the Combined Variable Equation and four variants of the Schumacher and Hall model [7] to estimate the stem volume outside bark of *P. occidentalis* trees in the Combined Intermediate and Humid Zone within La Sierra, Dominican Republic.

		Fit Statistics					**Validation Statistics**				**Ranking**	
Model	**Variant Code**	**RMSE (Rank)**	**BIAS (Rank)**	**SSRR (Rank)**	**RVE (Rank)**	**AIC (Rank)**	**RMSE (Rank)**	**BIAS (Rank)**	**SSRR (Rank)**	**RVE (Rank)**	**Sum Rank**	**Overall Rank**
Model (2): Effect Variable D^2H	CV01	7.85E−02 (5)	6.11E−18 (2)	1.06E+00 (9)	6.31E−03 (9)	−2.67E+02 (8)	1.01E−01 (4)	−7.29E−02 (8)	2.22E+00 (1)	6.70E−03 (4)	50	7
	CV02	7.87E−02 (6)	−4.01E−18 (1)	1.01E+00 (7)	6.19E−03 (6)	−2.99E+02 (5)	1.03E−01 (5)	−7.06E−02 (4)	2.39E+00 (2)	6.84E−03 (5)	41	4
	CV03	7.89E−02 (7)	−1.29E−03 (7)	1.01E+00 (6)	6.17E−03 (5)	−3.01E+02 (3)	1.06E−01 (6)	−7.24E−02 (5)	2.56E+00 (3)	7.17E−03 (7)	49	6
	CV04	8.08E−02 (9)	−5.31E−03 (8)	1.00E+00 (5)	6.19E−03 (7)	−3.11E+02 (2)	1.14E−01 (9)	−7.27E−02 (6)	3.23E+00 (5)	7.81E−03 (8)	59	8
	CV05	8.07E−02 (8)	−7.43E−03 (9)	1.02E+00 (8)	6.21E−03 (8)	−3.00E+02 (4)	1.14E−01 (8)	−7.43E−02 (9)	3.10E+00 (4)	7.85E−03 (9)	67	9
Model (3): Effect Variables D, H	SH01	7.59E−02 (4)	−5.22E−04 (5)	9.56E−01 (1)	5.94E−03 (4)	−2.39E+02 (9)	9.49E−02 (2)	−6.17E−02 (2)	4.14E+00 (6)	6.02E−03 (2)	35	2
	SH02	7.55E−02 (1)	6.86E−04 (6)	9.57E−01 (2)	5.86E−03 (3)	−2.74E+02 (7)	9.86E−02 (3)	−6.53E−02 (3)	4.33E+00 (8)	6.41E−03 (3)	36	3
	SH03	7.56E−02 (2)	2.08E−04 (4)	9.65E−01 (4)	5.83E−03 (2)	−3.12E+02 (1)	9.43E−02 (1)	−6.14E−02 (1)	4.22E+00 (7)	5.97E−03 (1)	23	1
	SH04	7.56E−02 (3)	−9.23E−05 (3)	9.65E−01 (3)	5.81E−03 (1)	−2.98E+02 (6)	1.08E−01 (7)	−7.27E−02 (7)	4.75E+00 (9)	7.43E−03 (6)	45	5

CV01: Combined Variable function first variant; CV02: Combined Variable function second variant; CV03: Combined Variable function third variant; CV04: Combined Variable function fourth variant; CV05: Combined Variable function fifth variant; SH01: De-transformed S&H Model with Bias Corrected variant; SH02: S&H Nonlinear Model variant; SH03: S&H Weighted Nonlinear Model Assuming Exponent C variant; SH04: S&H Weighted Nonlinear Model Modeling Exponent C variant; H: Total Tree Height.

Table 5. Parameter estimates, corresponding confidence intervals, residual standard error, and the adjusted coefficient of determination obtained in fitting the five variants of the Combined Variable Equation and four variants of the Schumacher and Hall model [7] to estimate the stem volume outside bark of *P. occidentalis* trees in the Dry Zone, La Sierra, Dominican Republic.

		CV (Model (2))					S&H (Model (3))			
Parameters	**Statistics**	**CV01**	**CV02**	**CV03**	**CV04**	**CV05**	**SH01**	**SH02**	**SH03**	**SH04**
B0	Residual Est. Error	1.66E−02	2.79E−02	1.25E+00	6.07E+01	6.87E−05	1.48E−02	1.52E−02	3.16E−05	1.47E−02
	Adjusted R2	9.92E−01	9.93E−01	9.92E−01	9.92E−01	7.60E−01	9.93E−01	9.93E−01	9.93E−01	9.93E−01
	Estimate	1.59E−02	1.35E−02	1.34E−02	1.25E−02	1.29E−02	6.14E−05	5.81E−05	5.88E−05	5.84E−05
	Lower Bound 95% CI	5.54E−03	6.80E−03	6.36E−03	7.69E−03	7.66E−03	4.69E−05	4.29E−05	4.48E−05	5.84E−05
	Upper Bound 95% CI	2.63E−02	2.02E−02	2.05E−02	1.73E−02	1.81E−02	8.04E−05	7.85E−05	7.71E−05	5.84E−05
	Pr (>|t|) Bo	3.66E−03	2.43E−04	4.72E−04	6.80E−06	1.49E−05	5.59E−39	1.02E−07	1.00E−08	9.99E−09
B1	Estimate	3.44E−05	3.46E−05	3.47E−05	3.48E−05	3.48E−05	1.82E+00	1.78E+00	1.82E+00	1.81E+00
	Lower Bound 95% CI	3.33E−05	3.37E−05	3.36E−05	3.38E−05	3.38E−05	1.73E+00	1.67E+00	1.72E+00	1.81E+00
	Upper Bound 95% CI	3.54E−05	3.56E−05	3.57E−05	3.59E−05	3.58E−05	3.63E+00	1.89E+00	1.91E+00	1.81E+00
	Pr (>|t|) B1	1.44E−38	1.51E−39	1.11E−38	7.22E−39	5.88E−40	1.52E−30	1.07E−27	2.04E−30	2.97E−30
B2	Estimate						1.02E+00	1.08E+00	1.04E+00	1.04E+00
	Lower Bound 95% CI						8.79E−01	9.64E−01	8.95E−01	1.04E+00
	Upper Bound 95% CI						2.04E+00	1.19E+00	1.18E+00	1.04E+00
	Pr (>|t|) B2						3.88E−16	9.02E−20	1.75E−16	1.35E−16
C	Estimate					1.74E+00			2.00E+00	1.90E+00

Table 6. Parameter estimates, corresponding confidence intervals, residual standard error, and adjusted coefficient of determination obtained in fitting the five variants of the Combined Variable Equation and four variants of the Schumacher and Hall model to [7] estimate stem volume outside bark of *P. occidentalis* trees in the Combined Intermediate and Humid Zone, La Sierra, Dominican Republic.

		CV (Model (2))					**S&H (Model (3))**			
Parameters	**Statistics**	**CV01**	**CV02**	**CV03**	**CV04**	**CV05**	**SH01**	**SH02**	**SH03**	**SH04**
B0	Residual Est. Error	5.82E−02	4.81E−02	4.61E−02	3.27E−02	3.63E−02	6.13E−05	5.86E−05	5.67E−05	5.57E−05
	Adjusted R2	3.04E−02	2.55E−02	2.34E−02	1.65E−02	1.70E−02	4.63E−05	4.33E−05	4.28E−05	5.57E−05
	Estimate	8.60E−02	7.07E−02	6.88E−02	4.89E−02	5.55E−02	8.15E−05	7.92E−05	7.50E−05	5.58E−05
	Lower Bound 95% CI	6.27E−05	4.80E−05	1.03E−04	1.13E−04	2.98E−04	3.84E−96	1.36E−09	1.33E−10	1.07E−10
	Upper Bound 95% CI	3.13E−05	3.17E−05	3.19E−05	3.26E−05	3.25E−05	1.82E+00	1.79E+00	1.78E+00	1.79E+00
	Pr (>|t|) Bo	3.04E−05	3.07E−05	3.08E−05	3.15E−05	3.14E−05	1.74E+00	1.71E+00	1.70E+00	1.79E+00
B1	Estimate	3.23E−05	3.28E−05	3.30E−05	3.37E−05	3.36E−05	3.65E+00	9.72E−01	1.86E+00	1.79E+00
	Lower Bound 95% CI	5.65E−95	9.39E−92	1.66E−88	5.40E−90	2.43E−91	1.67E−75	4.29E−76	5.83E−75	3.44E−75
	Upper Bound 95% CI						1.01E+00	1.06E+00	1.08E+00	1.08E+00
	Pr (>|t|) B1						9.28E−01	1.87E+00	9.89E−01	1.08E+00
B2	Estimate						2.02E+00	1.14E+00	1.17E+00	1.08E+00
	Lower Bound 95% CI						5.59E−46	2.36E−48	2.71E−47	3.74E−47
	Upper Bound 95% CI					2.03E+00			2.00E+00	2.20E+00
	Pr (>|t|) B2	7.91E−02	8.08E−02	1.17E+00	1.29E+01	6.45E−05	7.59E−02	7.55E−02	6.80E−05	7.56E−02
C	Estimate	9.73E−01	9.69E−01	9.65E−01	9.67E−01	9.68E−01	9.73E−01	9.73E−01	9.74E−01	9.74E−01

In Figure 3, scatter plots of standardized residuals plotted against fitted values (panels a and b) and quantile–quantile plots (panels c and d) are shown as tools to check the constant variance and normal distribution hypotheses of the residuals in the Dry Zone and Combined Intermediate and Humid Zone using modeling variants SH02 and SH03. The standardized residuals plotted against fitted values show randomly clustered values around the 0 line for outer-bark volumes in the Dry Zone (a) and the Combined Intermediate and Humid Zone (b). The points around the line in the quantile–quantile plots remain relatively constant across all levels of the predictor variables, confirming the constant variance assumption and closely following the observed values, suggesting that these models capture the variability in the data well.

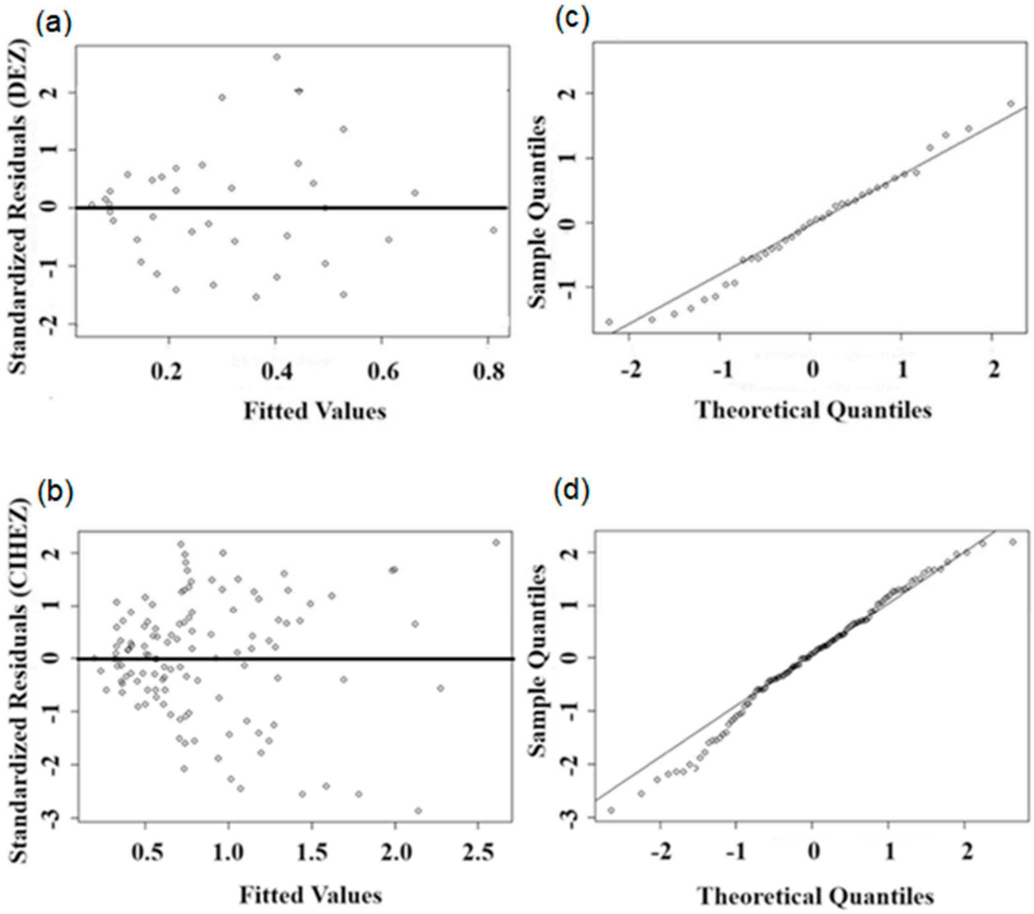

Figure 3. Residuals versus fitted values in the Dry Zone (**a**) and Combined Intermediate and Humid Zone (**b**) are shown on the left. Sample versus theoretical quantiles in the Dry Zone (**c**) and Combined Intermediate and Humid Zone (**d**) are depicted on the right.

In Figure 4, the top panel (a) shows predictions carried out by modeling variant SH02 against the volume predicted by the SH02 variant. Likewise, the bottom panel (b) shows the predictions carried out by modeling variant SH03 against the predicted volume in the Combined Intermediate and Humid Zone. These plots support evidence of the agreement between observations and predictions. SH02 and SH03 efficiently estimated the stem volume outside bark for *P. occidentalis* in both ecological zones and effectively captured the variability in the data.

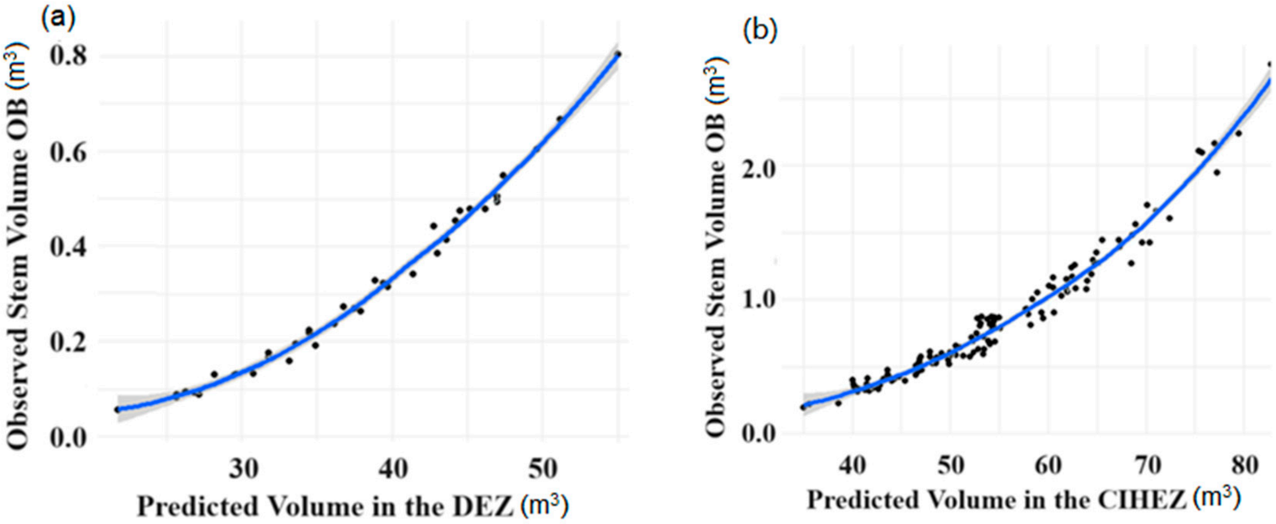

Figure 4. Predicted outside-bark volumes plotted against corresponding observed volumes in the Dry Ecological Zone (**a**) and Combined Intermediate and Humid Zone (**b**) are illustrated. SH02 and SH03 efficiently estimated the stem volume outside bark for *P. occidentalis* in both ecological zones and effectively captured the variability in the data.

4. Discussion

Alvarado-Segura et al. [17] used a nonlinear version of the SH model to estimate the stem volume of *Pinus patula* (Schl. et Cham.) in Hidalgo, Mexico, and reported an RMSE of 0.0914. As a measure of the average magnitude of the residual error, RMSE lower values indicate better goodness-of-fit of a regression model [18]. Our RMSE statistic value is much smaller and better for our SH02 modeling variant (0.0146), while the corresponding value for SH03 (0.0943) is close to that reported by these authors. The SH model has been found to perform well in predicting stem volume for many species and genera in diverse environments [19]. Castillo-López et al. [20] and Valerio-Hernández et al. [21] used this model in studying volume equations for pine species in Mexico and Nicaragua. Based on the evaluation criteria, this study's nonlinear version (SH02) and the nonlinear weighted variant (SH03) of the S&H model performed best.

All nine variants tested from models (2) and (3) in both EZs resulted in parameter estimates that are statistically significant and logically consistent for outside-bark volume estimation (Tables 5 and 6). The observed relationship between the dependent variable stem volume outside bark and the predictors is unlikely to have occurred by chance [18]. The estimates for intercept, interpreted as the average value of the total stem volume accumulated by a tree until it reaches breast height (when DBH is zero and total height is 1.3 m) were positive and significantly different from zero. All other parameters were consistent in terms of sign and magnitude.

The best-ranked Combined Variable model variant was CV02, which achieved an overall ranking of 3 in the Dry Zone and 4 in the Combined Intermediate and Humid Zone. Values for intercepting the coefficients of variants in model (2) resemble, in a certain way, the shape and form of a geometric solid, with higher numbers representing better and more cylindrical form, and thus, greater volume [12]. On average, model (2) variant intercepts are lower (105.77%) in the Dry Zone than their counterparts in the Combined Intermediate and Humid Zone, indicating that the stem volume accumulated by individual *P. occidentalis* trees below breast height is lower in the former zone and that trees are smaller. A *t*-test assuming unequal variances indicates that intercepts from the two zones are statistically and significantly different ($p = 0.0026$). The slope coefficients for D^2H in the same model (2) variants are also statistically and significantly different ($p = 0.000126$).

In mathematics, the volume of a circular base solid with base diameter D and height H is expressed as follows: $V = \beta D^2 H$ [22]. If the diameter is measured in cm, $\beta = \frac{\pi}{40{,}000}$

for a cylinder, $\frac{\pi}{80{,}000}$ for a paraboloid, $\frac{\pi}{120{,}000}$ for a cone, and $\frac{\pi}{160{,}000}$ for a neiloid. This β is equivalent to β_1, the coefficient in the second term of the CV. The averages of the five β_1 values from our results are equivalent to 0.0000347 in the Dry Zone, and 0.0000321 in the Combined Intermediate and Humid Zone, differing by about 12.4% and 20.47% off the perfect paraboloid, respectively. Similar values for β_1 using the CV equation in fitting data from 150 *P. patula* trees were found by Alvarado-Segura et al. [17] in Mexico. Therefore, tree shapes in our study approximate a paraboloid solid, which is described mathematically by the expression $\pi/80{,}000 = 0.0000393$ (diameters are in cm units). This contrasts with the results of Sharma [22] who found that the shape of the trees is not a solid described by a cylinder, paraboloid, cone, or neiloid while assessing the tree volume of twenty-five species in the natural stands of Canada and northeastern United States, including balsan poplar, eastern white cedar, Engelman Spruce and European larch. These are temperate forest trees, and the results may not be comparable, but it has been assumed that all trees should approximate one of these mathematical shapes, regardless of species.

Differences in the parameters of the CV models between EZs may result from differences in the height-dbh relationship, tree form, tree taper, or a combination of these factors [1], although all were logically consistent. These results agree with the findings reported by Sharma [23], who employed the CV equation to compare goodness-of-fit statistics, logical consistency, and the predictive accuracy of several models in *Pinus resinosa* Sol. ex Aiton. Exploratory analyses of variance not reported in the study showed that the form and quotient coefficients of *P. occidentalis* trees in these zones were statistically significant. That, combined with higher values for the β_0 coefficient in the Humid Zone, may indicate that the form is better there and, therefore, better suited for wood production.

5. Conclusions

Nine model variants, five from the CV and four from the SH, were evaluated in their capacity to estimate the stem volume outside bark (SVOB) of individual *P. occidentalis* trees growing in natural stands within three EZs in La Sierra, D.R. All the variants produced results consistent with those of other pine species. All modeling variants gave good results on the fitting data set, but the performance was poorer when used in the validation data set. Ercanli et al. [24] and Sahin [25] encountered the same situation while modeling the tree volume of conifer species in Turkey.

An indicator variable analysis indicated that only one equation was required to estimate the total stem volume outside bark in the Intermediate and Humid Zones. Therefore, observations from these two EZs were combined to evaluate the different modeling variants further.

The CV model variants were logically consistent in parameter estimation. They complied with regression assumptions but were inferior to the chosen SHVE model variants in terms of ranking based on goodness-of-fit statistics and predictive ability. Modeling variants SH02 and SH03, strategies for modeling V_{SOB} individual *P. occidentalis* trees, were efficient and selected as the volume equations for the Dry Zone and Combined Intermediate and Humid Zone, respectively.

The maximum bias among all modeling variants in the Dry Zone was 0.20% and 5.54% in the fitting and validation phases, respectively. Likewise, in the Combined Intermediate and Humid Zone, bias was 0.007% and 8.84%, respectively. According to Sharma [22], volume equations resulting in a bias larger than 10% are very imprecise, leading to poor performance and unreliable predictions. It should not be recommended for forest management decision-making. The minimum observed variability explained by all variants in both zones was 94% for outside-bark volume estimates.

Developing forest management strategies requires accurate tree volume estimates, employing tools such as tree volume models. These volume models are simpler to use if the entire stem volume is of interest [26]. The assumptions underlying the regression methods employed in the study for constructing these nine volume model variants were validated. The results showed that these modeling variants performed well in both the fitting and validation phases, and therefore, we propose their use as useful tools for predicting the total stem volume outside bark in these EZs within La Sierra, D.R. The equations presented here provide more scientifically accurate and consistent predictions of individual *P. occidentalis* trees. They are a valuable tool for supporting present and future management decisions.

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